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BERTRAND RUSSELL'S MATHEMATICAL EDUCATION

by

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INTRODUCTION

The influence of Bertrand Russell (F.R.S., 1909) on the foundations of mathematics has an assured place in history, but some mathematicians who look closely at this influence may well have doubts about Russell's overall view of mathematics and how compatible that view is with their own. The British mathematician R.O. Gandy, for example, has characterized certain of Russell's expressions as 'absurd' and 'stultifying' and as denying 'the significance of the mathematical imagination and the value of mathematical experience' (1). The French mathematician Jean Dieudonné has referred to Russell as one who 'pretends to have the reputation of being a mathematician and succeeded in doing so in the eyes of contemporaries (and even today in the eyes of a number of philosophers)' (2).

If Russell had a response, it might be conjectured from his summaries in an 1896 notebook of his reading of French authors on foundations of geometry. Russell grouped them under two headings, 'Fools' and 'Philosophers,' the latter group containing only the note 'None extant.' Among the six 'Fools' was Auguste Calinon about whom Russell wrote 'mere mathematician: doesn't care what philosophers think' (3).

We know of no evidence that Russell had pretensions to being a mathematician. However, someone who, like Russell, was ranked seventh Wrangler in the Cambridge Mathematical Tripos examinations in the late 19th century was probably entitled to believe that he had a good acquaintance with at least some mathematics. Some of the graduates of this 19th century system were among the most distinguished mathematicians of that century. There are two main points we wish to make. First, Part I of the Mathematical Tripos was the central part of an educational culture at Cambridge where mathematical knowledge was felt to be too important to be left to the mathematicians in spite of the fact that mathematicians decried the effect this had on the development of mathematics at Cambridge. Since the purpose of the Tripos was to train the mind, the content of the courses was regarded as less important than the facility with which the students learned to solve problems; nor was logical rigour prized, since physical intuition was insisted upon over abstraction. Second, it is historically useful to distinguish between Russell's mathematical education leading up to the Tripos and that which he continued beyond his student days. The former might be regarded as training and the latter as his more important education in mathematics. Though Russell subsequently made up for his retrograde mathematical training, the central place of mathematics in his philosophy is analogous to that in 19th century Cambridge: mathematics is more than just the product of what mathematicians do, for Russell it is also the key to understanding human knowledge. We hope that this account will help provide some background to that philosophy, especially in its early years leading to *The principles of mathematics* (1903)

RUSSELL'S PRE-CAMBRIDGE EDUCATION

Both Russell's parents had died before his fourth birthday in 1876 and he was brought up in rather isolated and repressive circumstances by his grandmother and a succession of tutors. For his earliest mathematical studies our main sources are memoirs written late in his life, as, for example, his *Portraits from memory and other essays*:

> My childhood was mainly solitary as my only brother was seven years older than I was. No doubt as a result of much solitude I became rather solemn, with a great deal of time for thinking but not much knowledge for my thoughtfulness to exercise itself upon. I had, though I was not yet aware of it, the pleasure in demonstrations which is typical of the mathematical mind. After I grew up I found others who felt as I did on this matter. My friend G.H. Hardy, who was professor of pure mathematics, enjoyed this pleasure in a very high degree. He told me once that if he could find a proof that I was going to die in five minutes he would of course be sorry to lose me, but this sorrow would be quite outweighed by pleasure in the proof. I entirely sympathized with him and was not at all offended.(4)

Russell went on to describe how his brother, Frank, introduced him to Euclid and how disappointed he was that the axioms, on which everything subsequent depended, could not be proved. Elsewhere he described his initial difficulties with algebra, which caused his tutor to fling the textbook at him (5).

During his adolescence, when Russell's interests were, as he put it, divided between sex, religion, and mathematics, his formal education continued at home with tutors (6). The tutors employed for him and his brother were not undistinguished. One, Douglas Spalding, a former student of the psychologist Alexander Bain, conducted experiments while employed by Russell's parents which have earned him a place in the history of ethology (7). Spalding was Frank Russell's tutor and Bertrand was too young to benefit from this association. But another he did benefit from was J.F. Ewen, an acquaintance of Karl Marx's daughter Eleanor Marx, who introduced Russell to both non-Euclidean geometry and Marxism (8). A third, J. Stuart, had been a student of Lord Kelvin (9). Tutors rarely stayed long, however, most being dismissed after a few months for fear they would undermine the influence of Russell's grandmother, who had her own plans for Russell and brooked little opposition to them. Ewen, who was apparently dismissed for fear he was undermining Russell's religious faith, was perhaps the most congenial for Russell. But Stuart also seems to have had some sympathy with the boy's intellectual interests, for, on New Year's Day 1888, he gave Russell a copy of the second edition of W.K. Clifford's *Common sense of the exact sciences*. Russell read the book 'with passionate interest and with an intoxicating delight in intellectual clarification' (10). Clifford's rather sketchy accounts of the basic concepts of arithmetic did not satisfy Russell for very long. But apparently they did save him from adopting John Stuart Mill's view that the truths of arithmetic were merely well-confirmed inductive generalizations. For when Russell came to read Mill's *System of logic*, just before he went up to Cambridge, he entirely rejected Mill's treatment of arithmetic, despite the fact that he was otherwise much influenced by Mill's opinions (11). Clifford seems to have been an important influence in this.

When Russell was 16 he was sent to the crammers, a school (University and Army Tutors in Southgate, London, directed by B.A. Green) whose main object was to prepare boys for the Army examination, but which was to prepare Russell for the scholarship examination at Trinity College, Cambridge. After a year and a half at the crammers he took the examination in December 1889 and obtained a scholarship to Trinity College. Apparently his preparation at the crammers was not the best – his *Autobiography* gives the impression that this first exposure to the world outside his home was more of a distraction from studies. His examination performance may also have been affected by his great nervousness and shyness at Cambridge (12). Alfred North Whitehead (F.R.S., 1903) was one of the examiners, along with H.M. Taylor and A. R. Forsyth, F.R.S., and Whitehead is given credit for seeing Russell's talent in spite of what was apparently not exactly a brilliant display. In fact Russell has said that 'another man had obtained more marks than I had, but Whitehead had the impression I was the abler of the two. He therefore burned the marks before the examiners' meeting and recommended me in preference to the other man' (13). As a result Russell was awarded the higher scholarship, its practical value being not the money but the status it afforded.

Between the time of the scholarship examination and his entrance into Trinity in October 1890, Russell was given further tutoring in mathematics at home. Mathematics was not an effortless matter. We have the first specific details, although still only hints, of his mathematics education from this interim cramming period. On his eighteenth birthday, 18 May 1890, Russell started what he described as 'a locked diary which I very carefully concealed from everyone' (14). The diary covers a wide range of matters and, though mathematics cramming was taking up a good deal of his time, there are only scattered minor references to it. Nevertheless these provide rather intriguing glimpses of his struggles with the obligatory exercises. The following is a sample:

20 May 1890. In the morning I worked at the Rigid but only got one example out. In the afternoon, ditto. ... The book [Ruskin's *Modern painters*] is most interesting to me as a study of mind; his mind is so exactly the antitype of the mathematical that I have great difficulty in entering into it. He has a certain artistic want of logic and sturdiness, which latter I should think must be almost inseparable from such a stationary pursuit as art appears to be, if not retrograde.

29 May 1890. In the morning read Routh (15), the Chapter on Motion in three Dimensions, with more comprehension than before, and got out several examples. I am really beginning to get a certain amount of grip of the subject.

7 June 1890. Yesterday went to Robson (16) in the morning, and when I got home began a Conics paper, which I finished this morning, having only done six out [of] eleven, but those six were the hardest. ... This morning drove in the pony carriage and did a very nice problem about vis viva and angular momentum. This afternoon did several easier ones....

16 June 1890. In the afternoon spent about two hours on a Rigid problem, which I spent three hours on again this morning, and finally did not solve, because I could get no first integral of my equations except the energy equation, and two were necessary for the solution.

20 June 1890. Wednesday, Robson, who showed me that my Sunday's Rigid Problem required no first integral at all, being an initial $\rho = y^2/2x = (\eta^2 t)/\xi$ if η, ξ be initial accelerations.

25 June 1890. ...almost all the problems I had to do came out by Lagrange's equations, which are very agreeable for examples in small oscillations (17).

In July Russell notes that he is no longer going to keep a regular journal 'but only to write now and then'. Though the diary continues to October 1894, after Russell left Cambridge, there are no more such mathematical details and far fewer entries after his arrival at Cambridge. During the period of the diary he meets, falls in love with and becomes engaged to Alys Pearsall Smith whom he married in December 1894. He is elected to The Apostles. He takes the Moral Sciences Tripos, Part II, and considers whether to take up philosophy, economics or politics, if not all three together.

The shift in the frequency and nature of diary entries is perhaps also coincident with a change in Russell's view of mathematics. However sceptical he may have been about the truth of the mathematics he had been learning, he appears to have had doubts of another nature about mathematics after arriving at Cambridge, at least about the effect of the sort of mathematical education he was undergoing. The last mention of mathematics in his diary is at the end of his first term:

15 December 1890 (Cambridge), The most curious character I have come across is Gaul (18). Though a wonderful mathematician, he is in every other respect vulgar and childish and materialistic and selfish. It is a pity to see such a person brought to the fore by the cram system, and I think an extension of the system introduced in the Scholarship Essay and General Paper would be a great advantage. A certain percentage in these two papers should I think be necessary no matter how well a man might do in his special subject.

> This would prevent the ignorant uncultured specialist from succeeding in the way he does now and would also prevent overwork to a great extent and encourage general intelligence, which is to my mind far more useful than great knowledge entirely restricted to one subject.

THE LACK OF HISTORICAL SOURCES

Russell was not a great keeper of memorabilia, but he was not an assiduous destroyer either. The one exception to the latter generalization concerns his mathematical training at Cambridge between 1890 and 1893. For that period we have virtually no personal documentation. His lecture notes, assignments, exam papers, are all lost – with the solitary exception of a revision notebook, dating from the May Term of 1893, which was preserved among the papers of Russell's second wife, Dora. Though it faithfully reflects the Tripos, and gives a clear idea of the sort of performance expected of a Wrangler, it is concerned exclusively with answering Tripos problems and tells us little directly about the scope or content of Russell's mathematical education (19). Russell even sold his mathematics books when he graduated (20). His reading list for the period does not include mathematical textbooks though it did include his reading for the Moral Sciences Tripos (21). Nor have letters dealing with his mathematics education survived from this period except for one to his uncle cited below. We have, from his autobiographical writings, a number of general remarks about how bad mathematics was in Cambridge in the early 1890s. But very little else. We don't even know for sure which subjects Russell studied, apart from those covered in the revision notebook. A long time later he wrote to his daughter – to encourage her in her studies – that he had had to study geometrical optics and spherical astronomy 'two subjects of incredible dullness' (22). We also know from stray reminiscences that he did a statics course with Whitehead and a course on hydrostatics (23).

The absence of documentation for Russell's years as a mathematics student is unique in his career. It means that if we are to learn very much about Russell's mathematical education we have to piece it together from such scattered and infrequent remarks as these, together with what we know of the situation at Cambridge at the time Russell studied there.

THE CAMBRIDGE MATHEMATICAL TRADITION

There was no strong connection between Russell's family and Cambridge. It is true that Russell's father had been there, but he had been sent down without taking a degree, and Russell's brother had gone to Oxford. Russell was drawn to Cambridge by its distinguished reputation for mathematics. From Newton through to Arthur Cayley, F.R.S., Cambridge had indeed a distinguished record in the history of English mathematics. Its reputation in the natural sciences, given a similarly powerful start by Newton, and considerably revitalized in the 19th century by

William Whewell, was comparably strong. Yet it was Russell's misfortune, going up in 1890, toward the end of Cayley's long reign as Sadleirian Professor of mathematics, that, as far as pure mathematics was concerned, Cambridge was in the doldrums, and lagged far behind its continental competitors such as Berlin and Göttingen.

Newton's heritage, which instilled so strong a sense of superiority at Cambridge, had become a prison for its 19th century mathematicians. Insular pride had originaly prevented the university from abandoning Newtonian methods, with their heavy use of physical and geometrical intuition, even though in the early 19th century more powerful and rigorous methods had been developed elsewhere, especially in France, in the form of abstract algebraic analysis. Contemporary analytic methods had been introduced to Cambridge, largely through the efforts of Robert Woodhouse and, in 1812, The Analytic Society organized mainly by George Peacock, F.R.S., Charles Babbage, F.R.S., and John F.W. Herschel, F.R.S. By the 1830s and 1840s analysis had found its place on the Tripos, with the result that the system produced during those years some of Cambridge's best pure mathematicians of the 19th century: J.J. Sylvester (1837) and Arthur Cayley (1842), among others.

This budding of Cambridge mathematics was curiously short-lived, and ironically it ended because of the dominant role that mathematics played in the Cambridge education system (24). Until the second half of the 19th century honours in the Mathematical Tripos was required for all degree candidates at Cambridge. Even after 1851 a pass in a part of the Mathematical Tripos was required. The contemporary ideas making their way into Cambridge under the patronage of The Analytical Society made a choice necessary concerning the aims of mathematical education. Short of doing away with the Tripos system altogether the University could design it for students intending careers as mathematicians or continue to use it as the central part of a liberal education for all. The decision was made by William Whewell, who became Master of Trinity College in 1841 and achieved thereby a position of unassailable authority within the University until his death in 1866. Ironically, Whewell had earlier been something of a supporter of the new analysis at Cambridge. But as a University administrator he turned his back resolutely on the new developments to pursue his vision of applied mathematics as the central part of a liberal education. In practice this meant: 'Back to Newton!' Forsyth noted the results:

> In patriotic duty bound, the Cambridge of Newton adhered to Newton's fluxions, to Newton's geometry, to the very text of Newton's *Principia*: in my own Tripos of 1881 we were expected to know any lemma in that great work by its number alone.... Thus ... Cambridge became a school that was self-satisfied, self-supporting, self-content, almost marooned in its limitations (25).

THE TRIPOS

However depressing a picture Forsyth paints, Russell and his fellow students came to Cambridge with the same sort of optimism which probably also guided Whewell and other architects of the Cambridge curriculum: 'The world seemed hopeful and solid; we all felt convinced that nineteenth-century progress would continue, and that we ourselves should be able to contribute something of value' (26). The confrontation between such confidence and the pernicious aspects of Cambridge education, epitomized by the Mathematical Tripos, was a major challenge for Russell.

The Mathematical Tripos in Russell's day was governed by regulations enacted by the University in the 1880s which had split the Tripos into two parts, elementary and advanced, both highly competitive (27). Part II of the Tripos covered a wide range of topics at an extremely advanced level from which students were expected to select a few areas of specialization. The new arrangements were widely criticized within the university (28). The difficulty of Part II, to which most areas of higher mathematics were consigned, ensured that these areas were studied by only a small group of specialists, while even Part I contained too many technical subjects to have the general appeal that was intended. Even partisan defenders of the system like Ball (29) had to admit that the number of students studying mathematics at Cambridge suffered a serious decline until sweeping reforms, of a sort which for the most part had Russell's approval, were enacted in 1907. Russell was one of those who did not take Part II in mathematics – instead he took up Moral Sciences – and thus it is only Part I of the Tripos that concerns us here.

Part I could be taken in the May and June of the student's second year in residence, but the highly competitive nature of the examination ensured that even the best students usually postponed it until their third year. Seven papers were set for the first four-day period in May:

- — pure geometry, and Newton I (Euclid, conic sections, *Principia*, Bk I, Section 1, in the 1883 edition by Percival Frost);
- — algebra and trigonometry (binomial theorem, exponentials, logarithms);
- — analytical geometry, differential and integral calculus (straight line, circle, reference to rectangular axes, parabola and hyperbola referred to principal axes, differentiation and integration of simple functions, Taylor's Theorem, maximum and minimum with one independent variable, tangents to curves, curvature, areas of curves);
- — statics, dynamics and Newton II, III (*Principia*, Bk I, Sections 2, 3);
- — optics and astronomy (without spherical trigonometry);
- — hydrostatics, heat and electricity (Ohm's laws, sine and tangent galvanometers).

More difficult material was reserved for the second group of examinations which was taken over a four-day period in June after an eleven-day break and only by those who did well enough in the first four days. The paper titles were not explicit: Natural Philosophy, Pure Mathematics, Pure Mathematics, Natural Philosophy, Pure Mathematics and Natural Philosophy, Pure Mathematics and Natural Philosophy, and finally Problems. Among the topics listed in *The student's guide* were: theory of equations, Fourier's theorem and calculus of variations, partial differential equations of first order, elliptic functions, hydrostatics including capillarity and rotating homogeneous liquid ellipsoids, dynamics of a particle, hydrodynamics including irrotational motion of a perfect fluid, two-dimensional waves and spherical astronomy.

For pure geometry and Newton I, and for specified problems in other subjects of the first set, 'elementary methods' only were to be used, which meant the problems must be solved 'without the aid of the elaborate machinery supplied by Modern Analysis.' That is, purely geometrical solutions were required, 'in some cases a much more difficult task than working out an Analytical solution' (30). This represented a change over previous Triposes and, though seeming only to reword instructions without necessarily affecting the examination syllabus, it is nevertheless a significant further opening of the door to increased use of the calculus and analytic geometry, tools which heretofore could only be used when explicitly allowed. This particular reform appears to have been initiated before Russell came up to Cambridge as an outcome of the results obtained in the 1888 Tripos. The Moderators and Examiners for the Tripos of that year reported to the Special Board for Mathematics that the dominance of elementary methods was forcing talented people to spend all their time on them and that a better distribution of problems was needed (31). Subsequent minutes of the Board reflected a debate over how much differential and integral calculus and analytic geometry was to be allowed in which parts of the examination. In particular, the Board at its meeting of 16 November 1889 passed several resolutions which appear to have led to the revised instructions for Russell's 1893 Tripos (32).

The range of subjects covered even by Part I was very wide, and students were warned against attempting 'too large a range of reading' and advised 'that any subject, or portion of a subject, which is undertaken, should be studied closely and thoroughly' (33). Nonetheless, since the final mark was the average of the marks obtained in all sections, this advice was not always heeded, and students acquiring a wide, superficial knowledge in the interests of accumulating marks became one of the besetting problems of the system. Given the educational objectives of the system, this could hardly have been unanticipated.

The accumulation of marks in the Tripos was extremely important. Russell would probably have agreed with Leslie Stephen's characterization from a don's point of view: 'Our plan is not to teach anyone anything, but to offer heavy prizes for competition in certain well-defined intellectual contests' (34). All candidates were ranked in strict order of merit, and classified in four divisions: Wranglers, Senior

Optimes, Junior Optimes and Pollmen. There were, of course, no pollwomen. Despite the existence of women's colleges, the women who lived in them were not members of the University and were not allowed to sit the University examinations without special permission until 1881. Even then they didn't achieve official standing in the Tripos. The reform of 1881 followed upon the publicity given Charlotte Angas Scott, a student of England's most renowned mathematician of the time, Arthur Cayley, when she was granted permission to sit the examination in 1880 and did as well as the eighth-ranked Wrangler. Russell read her book on analytic geometry in 1899 and she attended his lectures on the foundations of geometry at Bryn Mawr, where she taught, in 1896 (35).

THE MATHEMATICAL COACHES

The curiously antiquated mathematical syllabus was complemented by the even more curious teaching arrangements of the University. The official system of college instruction had long been inadequate to the needs of students who needed to do well in the Tripos. Lectures on mathematics were not intended to prepare the student specifically for the Tripos. Instead success demanded long hours of coaching in the types of problems to be found in the examination. This instruction was not provided by the Colleges but by private coaches who were often dons earning some money on the side, though sometimes they were not members of the University at all. They were paid for their coaching directly by their students, of whom they often had considerable numbers (36). The coach was responsible for directing his students' work and supervising their progress through the Tripos curriculum. This was achieved by lectures, often several times a week, to small groups of students and the setting and marking of large numbers of examples based on previous Tripos papers. Sample solutions to set problems would be given at the lectures, or circulated among the students in manuscript. Because many coaches were entirely dependent upon their students' fees, they found it expedient not to publish textbooks which would enable the students to dispense with their services. Instead the coaches 'codified mathematical knowledge into small tracts or pamphlets, kept in manuscript as [their] own private prescription for [their] own set of students' (37). A consequence of this privatization of knowledge was a serious dearth of good text-books, which began to be remedied only in the last years of the 19th century.

The most famous, and by far the most successful, of the coaches was E.J. Routh, F.R.S., Fellow of Peterhouse and the author of the Tripos text on rigid dynamics which Russell had studied before going up to Cambridge. Between 1858 and his retirement from coaching in 1888, Routh coached over 600 students, most of whom became Wranglers, 27 of them Senior Wranglers (including 24 in 24 consecutive years). He saw his students in groups of between six and ten for an hour three times a week. At the height of his powers he was teaching up to 120 students at a time, starting his first class at 7.15am. He kept this up through three terms and the Long Vacation. A full course of Routh's coaching consisted of ten terms and three Long Vacations. In a busy term he would be teaching till ten at night. At each class he would give out about six problems, the solutions to which would be dealt with at the beginning of the next class, the remainder of the time being devoted to exposition of the topic under consideration. Each week, in addition, he set all his students 12 problems selected from past Tripos papers. One week they would have unlimited time to solve the problems; the next they would have to try to complete the paper in three hours (the duration of a Tripos paper). Routh's labours were at least as arduous as those of his students: answers would be left at his rooms on Friday afternoon or Saturday morning and the following Monday morning he would leave the marked papers in an outer room to be collected, together with a set of solutions and a list of the names and marks of all the students. Forsyth, who was his student (and his bitter opponent when it came to reforming the Tripos system), says tactfully of his teaching:

> It was superbly direct for the purpose in view: and it was strong in the measured completeness with which he covered the whole ground for the Tripos. Independence on the part of the student was not encouraged; for independence would rarely, if ever, be justified by the event. Foreign books were seldom mentioned: Routh himself had summarized from them all that could be deemed useful for the examination (38).

Routh retired from coaching two years before Russell went up, but his methods epitomize the pedagogy that the Tripos brought forth, and other coaches (Russell's included) strove to emulate Routh's success. Russell's coach was Robert Rumsey Webb of St John's College, who had been Senior Wrangler in the Tripos of 1872, for which he had been coached by Routh. With Routh's retirement he became 'the most famous mathematical coach of his time' (39). Much less is known about him, but he must have been successful, since at one time he was teaching 60 hours a week. Forsyth described him as 'a superb teacher' (40), and '[t]he breadth and exactness' of his knowledge were said to be 'a source of wonder to his pupils' (41). But not, apparently, to Russell, who seems to have taken a course on hydrostatics with him and found it 'quite uninteresting' (42). Webb's obituary in *The Cambridge Review* diplomatically states:

> [I]f Routh was mild, and Besant (43) courteous, Webb was impressive. Those who enjoyed his teaching, and endured his personalities, learnt from him much more than mathematics; slovenliness, sham and personal advertisement received a decisive reward, in words not easily forgotten. But his devotion to his pupils ... shewed itself in a changed manner ... in after life.

Webb lived most of his life in College, in his later years as a virtual recluse. According to Russell, 'He went mad, but none of his pupils noticed it. At last he had to be shut up' (44). It is difficult to see what influence, if any, this somewhat curmudgeonly product of the Tripos system had on Russell.

Russell's surviving Tripos revision notebook (45) interestingly enough, does not contain notes from Webb, but from R.A. Herman. Herman, a Fellow of Trinity with interests in crystallography and geometrical optics, later coached J.E. Littlewood, who described him as 'the last of the great coaches' (46), though the general impression seems to have been that he was not the equal of Routh or Webb. Russell probably copied the notes from a friend who coached with Herman.

The notes are keyed by topic headings and article numbers to the examination papers and possibly also to some text which is not specified: 'Elementary Optics II' and 'v. arts 13, 20,' for example. The topics covered are: elementary and advanced optics, elementary astronomy, Newton, hydrostatics, elementary dynamics, elementary statics, trigonometry, elementary algebra, and rigid dynamics. Original problems or 'riders' are the main concern but neither they nor the more rote questions to which they are appended appear in these notes.

Russell took the Tripos in May and June 1893. On 18 May, the second day of examinations, Russell turned 21 and inherited £20,000. He wrote to his Uncle Rollo during the break between the first and second series of papers:

> I have been so exceedingly busy working for my Tripos. I have had hardly any opportunity of comparing myself with the others in the 1st $3^1/2$ days, but so far as I can tell I have done about as I expected to do. I have still a week's hard work and then the 2nd $3^1/2$ days wh. is the really important part, the 1st being more or less elementary. I have no notion where I ought to be, except that I am not likely to be in the 1st 10 (47).

Nevertheless he was bracketed with another student, G.W. Caunt, as Seventh Wrangler. There were 106 men placed for the Part I, 30 of these as Wranglers. One of those above Russell was C.P. Sanger, bracketed second, a friend who became a barrister of some renown (48). Gaul, referred to so unfavourably in Russell's diary above, was ranked fourth. The first ten Wranglers averaged 1691 marks out of a possible total of 8221 marks for the whole of Part I. Although this is not drastically different from the results for previous Triposes, the Moderators and Examiners commented that 'The quality of the work sent up by the candidates with one exception was disappointing especially in the new subjects.' Suggestions for improvements, they reported, might be made after the new scheme was tried further (49).

MATHEMATICS IN RUSSELL'S CAMBRIDGE

There were first-rate mathematicians at Cambridge in Russell's day, notably Cayley, among pure mathematicians. However, Cayley was already in his seventies when Russell went up and suffering from a long terminal illness which caused him to cancel lectures for a whole term in 1893. But it seems unlikely that Russell (or any of the undergraduates) would have attended the lectures had they been given. The gulf between undergraduates and professors of Cayley's eminence, or indeed any of the senior dons, was very wide. University and College lecture courses were regularly given but rarely attended by students studying for Part I of the Tripos. Such lectures were thought to be irrelevant for the examination.

Russell was probably not dissatisfied with the emphasis on applied mathematics at Cambridge. He thought, in his 'Victorian optimism' that 'applied mathematics ... was more likely to further human welfare', a view in which he had been encouraged by reading Clifford's *The common sense of the exact sciences* (50). The University was certainly better served by its applied mathematicians, many of whom were of considerable eminence; for example, J.J. Thomson, F.R.S., J. Larmor, F.R.S., G. Stokes and J.C. Adams, F.R.S.

Still, G.H. Hardy (F.R.S. 1910), one of the best-known Cambridge mathematicians of the early 20th century, reported in 1926 that he remembered Russell 'telling me that he studied electricity at Trinity for three years, and that at the end of them he had never heard of Maxwell's equations.... and when I think of this I begin to wonder whether the teaching of applied mathematics was really quite so perfect as I have sometimes been led to suppose' (51). It would be the younger dons with whom Russell was most likely to come into contact; perhaps with A.E.H. Love, whose lectures were said to be 'extremely popular with students for their clarity, intelligibility and real efforts to enter into the students' point of view' (52). Love also had Webb as mathematical coach and acknowledged his debt to Webb in his book *Theoretical mechanics* (1898) which Russell reviewed (53). In 1900 Love was one of those opposed to the order of merit of the Tripos but was cited by Routh during the reform battles of 1906 as having joined the other side (to Routh the right one) on this issue since becoming a professor at Oxford (54).

Love was among the 'new dons' who had begun to appear in Cambridge in other disciplines in the 1870s and 1880s, but whose emergence in mathematics seems to have been delayed (55). Another would be Whitehead who gained his first teaching position at Trinity in 1884, as part of an unsuccessful attempt to regain for the Colleges the teaching role that had by default fallen to the coaches. Whitehead, Russell wrote,

> was extraordinarily perfect as a teacher. He took a personal interest in those with whom he had to deal and knew both their strong and weak points. He would elicit from a pupil the best of which the pupil was capable. He was never repressive, or sarcastic, or superior, or any of the things that inferior teachers like to be. I think that in all the abler young men with whom he came in contact he inspired, as he did in me, a very real and lasting affection (56).

Whitehead at this time, surprisingly in view of his later work, was regarded as an applied mathematician. His Fellowship dissertation was on Maxwell's *Electricity and Magnetism* and he lectured throughout his Cambridge years in applied mathematics (57). Nonetheless, his research interests, by the time Russell went up, had changed and lectures by him on non-commutative algebras (work which culminated in his *Universal algebra* of 1898) were scheduled for 1892 and 1893, though the latter may well have been cancelled (58). It seems very unlikely, however, that Russell would have attended these. We do know that he attended a course on statics given by Whitehead during Michaelmas Term 1890. Russell recalled: 'He told the class to study article 35 in the textbook. Then he turned to me and said "You needn't study it, because you know it already." I had quoted it by number in the scholarship examination ten months earlier. He won my heart by remembering this fact' (59).

The only fond remembrance of undergraduate mathematics that Russell later recorded relates to applied mathematics: 'I remember a sense almost of intoxication when I first read Newton's deduction of Kepler's Second Law from the law of gravitation. Few joys are so pure or so useful as this' (60).

Russell's experience with experimental, as supposed to mathematical, physics was not extensive. In 1898 Russell did some experimental work in the Cavendish Laboratories at Cambridge. His lab notebook has survived with pencil notes of not-very-well labelled readings of Wheatstone Bridges, ohm-meters and other standard instruments, which he apparently used in conducting elementary exercises verifying basic laws of electricity, magnetism and optics. It gives the impression of a rough book for Sixth-Form lab work. All we have in explanation of this episode are two references. The first is in a letter to his wife, Alys, written 12 March 1898: 'I must be off to the Labs now, for the last time *this* year, thank Heaven!' (61). The second explanation is in a letter to Philip Jourdain and it summarizes the path which eventually returned Russell to pure mathematics:

> I first read Cantor's work in 1896; I was not then convinced that it was valid. I then worked for some time at the principles of Dynamics; I went to the Cavendish Laboratory, and I studied Clerk Maxwell. Gradually I found that most of what is philosophically important in the principles of dynamics belongs to problems in logic and arithmetic (62).

But it was the poverty of his training in pure mathematics that told against Russell in the years immediately after graduation, and which cost him a major effort through the 1890s to make good. He recalled that the '"proofs" that were offered of mathematical theorems were an insult to the logical intelligence.' Elsewhere he cited the proof of the binomial theorem as an example (63). Forsyth, who has told of particles being made to describe fantastic arabesques never encountered outside the confines of an examination room, agreed:

> The algebra and the trigonometry (there was no 'analysis' in the modern sense) belonged to the old school, the very old school: the proofs, such as were current about the Binomial Theorem, would stir the explosive contempt of the critical mathematicians of today. Nor was the calculus any better in its foundations: I can remember a college question, set years after my student time, 'Define a function, and prove that every

function has a differential coefficient.' The marvel is that our blunders were not greater in number and more atrocious in quality (64).

Russell said that the overall effect of the Tripos system was to make him 'think mathematics disgusting. When I had finished my Tripos, I sold all my mathematical books and made a vow that I would never look at a mathematical book again', a vow broken within a year, however (65). In *Portraits from memory* he wrote:

> My teachers offered me proofs which I felt to be fallacious and which, as I learnt later, had been recognized as fallacious. I did not know then, or for some time after I left Cambridge, that better proofs had been found by German mathematicians.... I was encouraged in my transition to philosophy by a certain disgust with mathematics, resulting from too much concentration and too much absorption in the sort of skill that is needed in examinations. The attempt to acquire examination techniques had led me to think of mathematics as consisting of artful dodges and ingenious devices and as altogether too much like a crossword puzzle (66).
>
> The men who taught me at Cambridge were almost wholly untouched by the Continental mathematics of the previous twenty or thirty years; throughout my undergraduate time, I never heard the name of Weierstrass (67).

But the shameful truth was worse than he admitted. In an unpublished paper of 1896 he argued that the calculus depends upon the contradictory notion of an infinitesimal. He treated the derivative as a ratio that becomes meaningless at the limit, where it is necessary (absurdly) for the differential to be a positive quantity smaller than any positive quantity that can be assigned (68). Both limits and infinitesimals are rejected in favor of the method of indivisibles where the differentials represent small *finite* quantities (69). It is not just the work on Weierstrass which is missing from this account, but any appreciation of even early 19th century work on limits. In this case, enlightenment came within the year, when he read De Morgan's calculus text of 1842 which gave him a passable definition of a limit (70). It is hardly surprising therefore that he should have dismissed the work of Cantor when he first read it in 1896, and there enlightenment took much longer.

That same year he read Cantor's papers and took extensive notes adding critical remarks which indicated his perplexities (71). His initial notes indicated a wariness of Cantor's classification of infinite sets. What Russell had regarded as collections of different orders of infinity – for example the collection of natural numbers; 1,2,3,..., and the collection of pairs of natural numbers (n,m) – were put into one-to-one correspondence with each other by Cantor and thus regarded as of the same class or power. When it came to Cantor's pages on addition and multiplication of transfinite numbers Russell gave up: 'I don't understand a word of them.'

At later times – at least once in 1899 and again in 1900, for example – he returned to these notes and added comments to his comments which, in effect, rescinded his earlier criticisms. Cantor's papers, with their numerous elegant and clear proofs of novel and sometimes counter-intuitive results, provide in themselves something of an introduction to modern analysis.

THE BEGINNING OF RUSSELL'S MATHEMATICAL RE-EDUCATION

Russell's return to mathematics after recovering from the Tripos is represented by his decision to do his Fellowship dissertation on the foundations of geometry. His philosophy teacher James Ward was probably the major influence on him at this time in these matters. Russell wrote to his fiancée, Alys, on 10 June 1894:

> We [Ward and Russell] discussed a lot of possible subjects and finally seemed to fix on the Epistemological Bearings of Metageometry, which sounds well at any rate. ... I may write on the meaning and validity of the differential calculus instead, but I think that would be harder and less exciting.

We have no copy of his dissertation, which was on metageometry and was well received. But, after much revision it became *An essay on the foundations of geometry* (1897) (72).

It is not known what caused Russell to realize how out-of-date his mathematical knowledge was. We don't know of anyone to whom he might have shown his flawed papers and who might have discouraged him from trying to publish them. He has stated that he first heard of the significance of Weierstrass when he was in the United States in 1896 (73). The source could have been someone like Charlotte A. Scott, mentioned above as having attended his lectures at Bryn Mawr. Or, more likely, James Harkness, Professor of Mathematics at Bryn Mawr, or Frank Morley, Professor of Mathematics at Haverford College in Pennsylvania (later at Johns Hopkins University), both of whom also attended his lectures and were joint authors of *Introduction to the theory of analytic functions* which Russell read in March 1899 (74).

CONCLUSION

Russell's faith in human progress through the development of applied science, which sustained him early in his mathematics studies at Cambridge, weakened as his labours in preparation for the Tripos lengthened. In part, the reasons were philosophical. The mid-Victorian materialistic rationalism with which he entered Cambridge, and which underlay his recipe for human progress through scientific advance, was undermined by the newer, neo-Hegelian philosophy that he encountered at Cambridge and to which he contributed (75). This philosophy led him, until he rejected it by 1898, to seek other sources of human progress. But, even without it, he would have been hard pressed to retain much confidence that the sort of unremitting grind required for success in the Tripos could do much to equip him for making a contribution to human welfare. This in spite of the University's best intentions as sardonically put by Leslie Stephen:

> As a mere intellectual toy, mathematics is far ahead of any known invention. ... To have been in love with some women is, we are told, to have received a liberal education;.... A three years' flirtation with mathematics is supposed to produce the same effect. We may never meet them again, or meet them only to pass scrupulously by on the other side. But our minds have been strengthened and prepared for dealing with other subjects (76)

Russell's years at Cambridge preparing for the Mathematical Tripos left him with an abiding dislike of mathematics considered as a system of techniques for solving problems, or as a system of rules for the manipulation of symbols. It is true that the Tripos mathematics was resolutely applied mathematics. But its applications were unreal. The typical Tripos problem had no real physical relevance and no real significance for pure mathematics either. It was a puzzle with cunningly devised conditions, designed to test the computational abilities of the students. This experience must have reinforced his earlier belief that pursuing political economy, then taught at Cambridge as part of the Moral Sciences Tripos, was the best way to prepare for making a contribution to human welfare.

Russell's other reason for his early interest in mathematics was philosophical. From his adolescence onwards he hoped to discover how much could be known and with what degree of certainty. Scepticism had undermined first his religious faith. But, once started, he found sceptical doubt hard to quell, to the extent that it seemed likely to entirely undermine the whole fabric of human knowledge. After his arrival at Cambridge, Russell never seriously expected to recover his religious faith. Indeed faith was not what he was after. He wished to establish a system of belief, so rationally grounded as to be proof against sceptical attack. A natural place to begin this endeavour was with mathematics, which held pride of place, of all branches of human knowledge, not only as the most rational but the most secure against sceptical attack. Yet it is also clear that the mathematics Russell was taught for the Tripos would no more support this ambition than it would the other. The proofs given students were 'an insult to the logical intelligence.' Rigour was not merely lacking, but actively avoided as an impediment to examination success. What Russell wanted from mathematics was not a set of techniques or results, but the starting point for the rational reconstruction of knowledge. The pursuit of this ambition also naturally led to the Moral Science Tripos, this time to the study of philosophy. Again Russell's adoption of neo-Hegelianism fitted well with these aspirations. For his friend, J.McT.E. McTaggart, was already claiming that a rigorous proof of a large metaphysical system was possible. And F.H. Bradley, the neo-Hegelian whom Russell admired most, was thought to have founded the Absolute by means of the most coruscating sceptical attack on virtually all forms of human knowledge.

It is not surprising therefore that in his fourth year at Cambridge, Russell studied Moral Science. He did not, at this stage, have to abandon either philosophy or political economy. The choice between them came in 1894 at the end of his second Tripos when he had to choose a topic for his Fellowship dissertation. Even then he considered, for a while, writing two dissertations: one on economics and one on the foundations of geometry. The choice of geometry was quite natural given his concern with human knowledge. For geometry in the 19th century was the centre of a strong foundationalist debate, occasioned by the development of non-Euclidean geometries. Euclidean geometry had appeared to be not merely necessary and certain, but also to be applicable in some way to the real world. This long-standing belief had been challenged by the development of the new geometries. It was altogether a natural place for someone with Russell's ambitions to start.

From here the future course of Russell's career began to develop until his well-known body of work was built up. Dieudonné was right in thinking that Russell was not a mathematician. He was not interested primarily in mathematics for its own sake, although on a number of occasions he did record his pleasure in pure mathematical results, in particular theorems and certain kinds of proofs. Of course, many non-mathematicians experience the same kind of feelings. But these were not his professional concerns. He approached mathematics, throughout the years until the completion of *Principia mathematica*, always with a philosophical project in mind. Mathematics itself, he thought, had to be placed upon a rational foundation, not because there was a real danger that mathematical theorems might turn out to be false – Russell, at least later, always denied that this was a real danger – but because only when it were done would mathematics be fit to stand as the cornerstone for the whole edifice of human knowledge.

ACKNOWLEDGEMENT

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NOTES

(1) R.O. Gandy, 'Bertrand Russell as mathematician' *Bull. Lond. Math. Soc.*, 342–348 (1973).

(2) Jean Dieudonné, 'Louis Couturat et les mathematiques de son époque,' in *L'Oeuvre de Louis Couturat (1878–1914 ... de Leibniz à Russell)* (Paris: Presses de l'Ecole Normal Superieure, 1983), p. 107.

(3) The other names are: Renouvier, Lechalas, Delboeuf, Broglie, and Liard. 'French geometry' notebook, RA REC ACQ 1027. Material from the Russell Archives at McMaster University is referred to by its RA identification number.

(4) Bertrand Russell, *Portraits from memory and other essays* (London: Allen & Unwin, 1956), p. 19.

(5) Bertrand Russell, *The autobiography of Bertrand Russell* (3 vols.) (London: Allen and Unwin, 1967–69), vol. 1, p. 36. The algebra text is unknown, but the geometry text was S. Hawtrey, *An introduction to the elements of Euclid*, 3rd edn (London: Longmans,

Green, 1880). Frank Russell's diary (RA 741.080043) records the lessons but not Russell's balking at unprovable axioms. He also records his satisfaction with Russell's progress. Russell noted on a copy of a letter from Frank dated 4 October 1914 that John Colenso was 'the author of the textbook of arithmetic on which we were brought up'.

(6) Russell, *Autobiography* vol. 1, p. 38.

(7) Robert Boakes, *From Darwin to behaviourism. Psychology and the minds of animals* (Cambridge: Cambridge University Press, 1984), pp. 15–16.

(8) Note by Russell to a letter from Ewen, RA 710.049836.

(9) Stuart to Russell, 19 January 1922.

(10) Bertrand Russell, 'Preface' to W.K. Clifford, *The common sense of the exact sciences*, ed. Karl Pearson and J. R. Newman (New York: Knopf, 1946), p. v.

(11) Bertrand Russell, *Portraits from memory*, pp. 124–5, 'My Mental Development' in *The philosophy of Bertrand Russell,* ed. P.A. Schilpp (New York: Harper and Row, 1963), p. 8, and *Fact and fiction* (London: Allen and Unwin, 1961), p. 44.

(12) Russell, *Autobiography* vol 1, p. 56.

(13) Russell, *Autobiography* vol 1, p. 57. Victor Lowe conjectures that the burning is Mrs Whitehead's embellishment (Victor Lowe, *Alfred North Whitehead: The man and his* work vol. 1. (Baltimore and London: The Johns Hopkins University Press, 1985), p. 222).

(14) Russell, *Autobiography* vol. 1, p. 82

(15) E.J. Routh, *The elementary part of a treatise on the dynamics of a system of rigid bodies* (London: Macmillan, 1882) and his *The advanced part of a Treatise on the dynamics of a system of rigid bodies* (London: Macmillan, 1884).

(16) H.C. Robson, his tutor. Later Robson became a fellow of Sidney Sussex College. *The Times* noted in his obituary of 27 January 1945: 'As a teacher of mathematics he was thorough and painstaking, and his method was well adapted to the needs of all but the ablest men.'

(17) Bertrand Russell, *The collected papers of Bertrand Russell* (London: George Allen & Unwin, 1983) vol. 1, pp. 43–54.

(18) It seems likely that Gaul was misidentified as Percy Corey Gaul in Russell's *Collected papers* vol, 1, p. 394. It was probably Percy Corey's brother, Claud Alfred, to whom Russell refers here. C.A. Gaul's mathematical career was less distinguished than that of his brother, but he was Russell's exact contemporary at Cambridge – as fourth Wrangler, placing above Russell in the Tripos of 1893. He also placed well ahead of Russell in the Trinity scholarship examination of 1891 (Trinity College Cambridge, Archives, Rec. 47.1 Mathematics Examinations 1887 – 1968.)

(19) The notebook, described below, survived because two years later Russell used its blank versos to make notes on his reading on non-Euclidean geometry.

(20) Russell, *My philosophical development* (London: Allen and Unwin, 1959), p. 38.

(21) From 1891 to 1902 Russell kept a list of the books he read in a small book with the title 'What Shall I Read?'. The list is printed in *Collected papers*, vol. 1, pp. 347–365.

(22) Russell to Katharine Tait, 6 December 1946. Russell Archives.

(23) Russell, *Autobiography* vol. 1, p. 127; *Portraits from memory*, p. 59.

(24) The main sources for the general background a A. R. Forsyth, 'Old Tripos days at Cambridge,' *The Mathematical Gazette* **19**, 162–791 (1935); K. Pearson's reply, 'Old Tripos days at Cambridge, as seen from another viewpoint,' *The Mathematical*

Gazette, **20**, 27–36 (1936); W.W.R. Ball, *History of the study of mathematics at Cambridge* (Cambridge: Cambridge University Press, 1889); I. Grattan-Guinness, 'A mathematical union: William Henry and Grace Chisholm Young,' *Ann. Sci.* **29**, 105–82 (1972), and 'University mathematics at the turn of the century: unpublished recollections of W.H. Young,' *Ann. Sci.* **28**, 369–84 (1972); H.W. Becher, 'William Whewell and Cambridge mathematics,' *Hist. stud. phys. Sci.* **11**, 1–48 (1980); together with contemporary university publications such as *The student's guide* which gives detailed lists of books to be studied for the Triposes.

(25) A.R. Forsyth, 'Old Tripos Days,' p. 167.

(26) Russell, 'My Mental Development,' p. 9.

(27) University graces of 13 December 1883, 12 June 1884, 10 February 1885, 29 October 1885, and 1 June 1886.

(28) Attempts to reform the system in the 1860s and 1880s are described in D. A. Winstanley, *Later Victorian Cambridge* (Cambridge: Cambridge University Press, 1947), pp. 223–33.

(29) W.W R. Ball, *Cambridge notes, chiefly concerning Trinity College and the University* (Cambridge: Heffer, 1921), p. 302

(30) *The student's guide*, p. 15.

(31) Cambridge University Archives, Special Board for Mathematics, Min. V.7. The Moderators and Examiners for 1888 were: W.H. Macaulay, W.B. Allcock, J. Larmor, and W. Welsh.

(32) Min V.7. At the meeting, those on the side of minimal reform appear (from the resolutions they moved or seconded) to have been E. J. Routh (described below), Mallison, E.W. Hobson, and Frost, while those calling for change were Forsyth, G.H. Darwin, and Macaulay. Also at the meeting were: N.M. Ferrers, J. Larmor (Chair), Webb, Allcock, Welsh, and Gallop.

(33) 'The Mathematical Tripos' in *The student's guide to the University of Cambridge, Part I*, 5th edn rev. (Cambridge: Leighton Bell, 1893), p. 14.

(34) *Sketches from Cambridge, by a Don*, (London and Cambridge: Macmillan and Co., 1865), p. 131.

(35) Patricia Kenschaft has gathered many details about Scott and her career, principally in her article on Scott in L.S. Grinstein & P.J. Campbell, (ed.), *Women of mathematics: a bibliographic sourcebook*, (New York: Greenwood Press, 1987).

(36) W.H. Young, for example, saved £6000 during six successful years coaching from 1890 to 1896. He managed this by teaching two groups of students in different rooms at the same time, alternating between them to give instructions. Grattan-Guinness, 'University mathematics,' pp. 114–5. For more details of the coaching system see Ball, *Cambridge notes*, pp. 303–306 and Stephen, *Sketches from Cambridge*, Ch. IX.

(37) Forsyth, 'Old Tripos days,' p. 167.

(38) A.R. Forsyth, 'Edward John Routh,' *Proc. Lond. Math. Soc.* **5**, xvi (1906–7).

(39) 'Obituary. R.R. Webb,' *The Eagle* (St. John's College), **49**, 273 (1937).

(40) Forsyth, 'Old Tripos Days,' p. 168.

(41) 'Robert Rumsey Webb,' *The Cambridge Review* **58** (No. 1412, 16 October 1936), 25.

(42) Russell, *Portraits from memory*, p. 60.

(43) William Henry Besant (1828–1917), Senior Wrangler 1850, was a Fellow of St. John's and one of the well-known mathematical coaches.

(44) Russell, *Portraits from memory*, p. 57.
(45) 'Herman notebook,' RA REC ACQ 1027.
(46) J.E. Littlewood, *Littlewood's miscellany*, ed. B. Bollobas, (Cambridge: Cambridge University Press, 1986), p. 83.
(47) Russell to Rollo Russell, 21 May 1893.
(48) Russell, *Autobiography*, vol. 1, p. 68. Webb, like many other coaches, used to circulate solutions in manuscript among his students. It was in delivering one such manuscript that Russell met Sanger (*Autobiography*, vol. 1, p. 57).
(49) Cambridge University, Archives, Special Board for Mathematics, Min V.7. The Moderators were E.J. Routh, and W. Burnside; the Examiners J.M. Dodds, and E.G. Gallop. The one exception was presumably the Senior Wrangler, G.T. Manley.
(50) Russell, *My philosophical development*, p. 39; Russell, 'Preface' to Clifford *The common sense of the exact sciences*, p. 14.
(51) G.H. Hardy, 'The case against the Mathematical Tripos,' *The Mathematical Gazette* **13**, pp. 61–71 at p. 65 (1926); *Collected papers* (Oxford: Oxford University Press, 1979) vol. 7, pp. 527–38.
(52) E.A. Milne, 'Augustus Edward Hough Love,1863–1940,' *J. Lond. Math. Soc.* **16**, 72 (1941).
(53) *Mind*, n.s., **7**, 404–411 (1898). *Collected papers* vol. 2, pp. 99–106 (London: Unwin Hyman, forthcoming).
(54) Cambridge University Library, Manuscripts, Kelvin Collection, Routh to Lord Kelvin 19 May and addendum of 20 May 1906, letter R112. 'Since he had been in Oxford [Love] had found that a man could not be made to work properly without it.' Kelvin provided choice ammunition for those resisting modernizations: '..the very best reform that could be made would be to go back to the Tripos Exam as it was in 1845 with only some modification in the way of physical applications' (draft letter Kelvin to Routh 22 May 1906, quoted in Routh to Kelvin 10 October 1906, R113, L102).
(55) Cf. Sheldon Rothblatt, *The revolution of the Dons* (London: Faber, 1968).
(56) Russell, *Portraits from memory*, p. 99.
(57) Lowe, *Alfred North Whitehead*, p. 311–12.
(58) *Cambridge Reporter* 6 October 1892, and subsequent issues.
(59) Russell, *Autobiography*, vol. 1, p. 127.
(60) Russell, *Education and the good life* (New York: Liveright, 1926), p. 259.
(61) Some explanation of his distaste may be indicated in a letter written at the beginning of 1914 when he began a rather intense period of work, including the writing of 'The relation of sense-data to physics' and wrote to his mistress, Ottoline Morrell: 'I often wish I were a physicist, but laboratories & experiments & mechanisms baffle me completely' (18 January 1914).
(62) Russell to Jourdain, 15 April 1910, I. Grattan-Guinness, *Dear Russell–Dear Jourdain, a commentary on Russell's logic, based on his correspondence with Philip Jourdain,* (London: Duckworth, 1977), p. 132. Russell is leading up to a response to Jourdain's curiosity about Peano's influence on Russell.
(63) Cf. Alan Wood, *Bertrand Russell: the passionate sceptic* (London: Allen and Unwin, 1957), p. 26, and M.D. Howe, ed. *The Pollock-Holmes letters: correspondence of Sir Frederick Pollock and Mr. Justice Holmes 1874–1932,* (Cambridge: Cambridge University Press, 1942), Pollock's letter of 24 September 1912, 1, p. 203.

(64) Forsyth, 'Old Tripos Days,' p. 170.
(65) Russell, *My philosophical development*, p. 38.
(66) *Portraits from memory*, pp. 15–6.
(67) Russell, *Education and the good life*, p. 310. This last may not have been entirely the fault of Cambridge. The University did offer Weierstrass an honorary degree while Russell was an undergraduate.
(68) Russell, 'On some difficulties of continuous quantity,' *Collected papers* vol. 2, p. 50.
(69) Russell, 'Various notes on mathematical philosophy,' *Collected papers* vol. 2, pp. 13, 15n, 17; Russell, 'On some difficulties,' *Collected papers* vol. 2, pp. 49–50.
(70) Augustus De Morgan, *The differential and integral calculus* (London: Baldwin and Craddock, 1842), p. 27. See also Russell's note on the manuscript of 'On some difficulties' cited in *Collected papers* vol. 2, annotation to p. 50, lines 28–9, and his manuscript 'De Morgan's definition of a limit' (RA 220.010680).
(71) The 'French geometry' notebook, RA REC ACQ 1027, contains the notes on a series of articles by G. Cantor, originally published in German but read by Russell in French translation in *Acta math., Stockh.* **2**, 305–414 (1883). The notes on Cantor will be published in *Collected papers*, vol. 2, Appendix III.2.
(72) For some account of the revisions in so far as they can be inferred from existing manuscripts, see N. Griffin, *Russell's idealist apprenticeship,* (Oxford: Oxford University Press, in press).
(73) Russell, 'My mental development,' pp. 14–15.
(74) Russell, *Collected papers*, vol. 1, pp. 340, 362.
(75) See N. Griffin, *Russell's idealist apprenticeship*.
(76) *Sketches from Cambridge,* pp. 32–33.