

CONTENTS

Preface	xiii
Acknowledgments.	xv
The Authors	xvii
1. The Spaces \mathbb{R}, \mathbb{R}^k, and \mathbb{C}	1
1 THE REAL NUMBERS \mathbb{R}	1
Properties of the Real Numbers \mathbb{R} , 2. The Absolute Value, 7. Intervals in \mathbb{R} , 10	
2 THE REAL SPACES \mathbb{R}^k	10
Properties of the Real Spaces \mathbb{R}^k , 11. Inner Products and Norms on \mathbb{R}^k , 14. Intervals in \mathbb{R}^k , 18	
3 THE COMPLEX NUMBERS \mathbb{C}	19
An Extension of \mathbb{R}^2 , 19. Properties of Complex Numbers, 21. A Norm on \mathbb{C} and the Complex Conjugate of z , 24. Polar Notation and the Arguments of z , 26. Circles, Disks, Powers, and Roots, 30. Matrix Representation of Complex Numbers, 34	
4 SUPPLEMENTARY EXERCISES	35
2. Point-Set Topology	41
1 BOUNDED SETS	42
Bounded Sets in \mathbb{X} , 42. Bounded Sets in \mathbb{R} , 44. Spheres, Balls, and Neighborhoods, 47	
2 CLASSIFICATION OF POINTS	50
Interior, Exterior, and Boundary Points, 50. Limit Points and Isolated Points, 53	
3 OPEN AND CLOSED SETS	55
Open Sets, 55. Closed Sets, 58. Relatively Open and Closed Sets, 61. Density, 62	

4	NESTED INTERVALS AND THE BOLZANO-WEIERSTRASS THEOREM	63
	Nested Intervals, 63. The Bolzano-Weierstrass Theorem, 66	
5	COMPACTNESS AND CONNECTEDNESS	69
	Compact Sets, 69. The Heine-Borel Theorem, 71. Connected Sets, 72	
6	SUPPLEMENTARY EXERCISES	75
3.	Limits and Convergence	83
1	DEFINITIONS AND FIRST PROPERTIES	84
	Definitions and Examples, 84. First Properties of Sequences, 89	
2	CONVERGENCE RESULTS FOR SEQUENCES.	90
	General Results for Sequences in \mathbb{X} , 90. Special Results for Sequences in \mathbb{R} and \mathbb{C} , 92	
3	TOPOLOGICAL RESULTS FOR SEQUENCES	97
	Subsequences in \mathbb{X} , 97. The Limit Superior and Limit Inferior, 100. Cauchy Sequences and Completeness, 104	
4	PROPERTIES OF INFINITE SERIES	108
	Definition and Examples of Series in \mathbb{X} , 108. Basic Results for Series in \mathbb{X} , 110. Special Series, 115. Testing for Absolute Convergence in \mathbb{X} , 120	
5	MANIPULATIONS OF SERIES IN \mathbb{R}	123
	Rearrangements of Series, 123. Multiplication of Series, 125. Definition of e^x for $x \in \mathbb{R}$, 128	
6	SUPPLEMENTARY EXERCISES	128
4.	Functions: Definitions and Limits	135
1	DEFINITIONS.	135
	Notation and Definitions, 136. Complex Functions, 137	
2	FUNCTIONS AS MAPPINGS	139
	Images and Preimages, 139. Bounded Functions, 141. Combining Functions, 142. One-to-One Functions and Onto Functions, 144. Inverse Functions, 147	
3	SOME ELEMENTARY COMPLEX FUNCTIONS	148
	Complex Polynomials and Rational Functions, 148. The Complex Square Root Function, 149. The Complex Exponential Function, 150. The Complex Logarithm, 151. Complex Trigonometric Functions, 154	
4	LIMITS OF FUNCTIONS	156
	Definition and Examples, 156. Properties of Limits of Functions, 160. Algebraic Results for Limits of Functions, 163	
5	SUPPLEMENTARY EXERCISES	171

5. Functions: Continuity and Convergence	177
1 CONTINUITY	177
Definitions, 177. Examples of Continuity, 179. Algebraic Properties of Continuous Functions, 184. Topological Properties and Characterizations, 187. Real Continuous Functions, 191	
2 UNIFORM CONTINUITY	198
Definition and Examples, 198. Topological Properties and Consequences, 201. Continuous Extensions, 203	
3 SEQUENCES AND SERIES OF FUNCTIONS	208
Definitions and Examples, 208. Uniform Convergence, 210. Series of Functions, 216. The Tietze Extension Theorem, 219	
4 SUPPLEMENTARY EXERCISES	222
6. The Derivative	233
1 THE DERIVATIVE FOR $f : D^1 \rightarrow \mathbb{R}$	234
Three Definitions Are Better Than One, 234. First Properties and Examples, 238. Local Extrema Results and the Mean Value Theorem, 247. Taylor Polynomials, 250. Differentiation of Sequences and Series of Functions, 255	
2 THE DERIVATIVE FOR $f : D^k \rightarrow \mathbb{R}$	257
Definition, 258. Partial Derivatives, 260. The Gradient and Directional Derivatives, 262. Higher-Order Partial Derivatives, 266. Geometric Interpretation of Partial Derivatives, 268. Some Useful Results, 269	
3 THE DERIVATIVE FOR $f : D^k \rightarrow \mathbb{R}^p$	273
Definition, 273. Some Useful Results, 283. Differentiability Classes, 289	
4 THE DERIVATIVE FOR $f : D \rightarrow \mathbb{C}$	291
Three Derivative Definitions Again, 292. Some Useful Results, 295. The Cauchy-Riemann Equations, 297. The z and \bar{z} Derivatives, 305	
5 THE INVERSE AND IMPLICIT FUNCTION THEOREMS	309
Some Technical Necessities, 310. The Inverse Function Theorem, 313. The Implicit Function Theorem, 318	
6 SUPPLEMENTARY EXERCISES	321
7. Real Integration	335
1 THE INTEGRAL OF $f : [a, b] \rightarrow \mathbb{R}$	335
Definition of the Riemann Integral, 335. Upper and Lower Sums and Integrals, 339. Relating Upper and Lower Integrals to Integrals, 346	

2	PROPERTIES OF THE RIEMANN INTEGRAL	349
	Classes of Bounded Integrable Functions, 349. Elementary Properties of Integrals, 354. The Fundamental Theorem of Calculus, 360	
3	FURTHER DEVELOPMENT OF INTEGRATION THEORY	363
	Improper Integrals of Bounded Functions, 363. Recognizing a Sequence as a Riemann Sum, 366. Change of Variables Theorem, 366. Uniform Convergence and Integration, 367	
4	VECTOR-VALUED AND LINE INTEGRALS.	369
	The Integral of $f : [a, b] \rightarrow \mathbb{R}^p$, 369. Curves and Contours, 372. Line Integrals, 377	
5	SUPPLEMENTARY EXERCISES	381
8.	Complex Integration	387
1	INTRODUCTION TO COMPLEX INTEGRALS	387
	Integration over an Interval, 387. Curves and Contours, 390. Complex Line Integrals, 393	
2	FURTHER DEVELOPMENT OF COMPLEX LINE INTEGRALS	400
	The Triangle Lemma, 400. Winding Numbers, 404. Antiderivatives and Path-Independence, 408. Integration in Star-Shaped Sets, 410	
3	CAUCHY'S INTEGRAL THEOREM AND ITS CONSEQUENCES	415
	Auxiliary Results, 416. Cauchy's Integral Theorem, 420. Deformation of Contours, 423	
4	CAUCHY'S INTEGRAL FORMULA	428
	The Various Forms of Cauchy's Integral Formula, 428. The Maximum Modulus Theorem, 433. Cauchy's Integral Formula for Higher-Order Derivatives, 435	
5	FURTHER PROPERTIES OF COMPLEX DIFFERENTIABLE FUNCTIONS	438
	Harmonic Functions, 438. A Limit Result, 439. Morera's Theorem, 440. Liouville's Theorem, 441. The Fundamental Theorem of Algebra, 442	
6	APPENDICES: WINDING NUMBERS REVISITED	443
	A Geometric Interpretation, 443. Winding Numbers of Simple Closed Contours, 447	
7	SUPPLEMENTARY EXERCISES	450
9.	Taylor Series, Laurent Series, and the Residue Calculus	455
1	POWER SERIES	456
	Definition, Properties, and Examples, 456. Manipulations of Power Series, 464	
2	TAYLOR SERIES	473

3	ANALYTIC FUNCTIONS	481
	Definition and Basic Properties, 481. Complex Analytic Functions, 483	
4	LAURENT'S THEOREM FOR COMPLEX FUNCTIONS	487
5	SINGULARITIES	493
	Definitions, 493. Properties of Functions Near Singularities, 496	
6	THE RESIDUE CALCULUS	502
	Residues and the Residue Theorem, 502. Applications to Real Improper Integrals, 507	
7	SUPPLEMENTARY EXERCISES	512
	10. Complex Functions as Mappings.	515
1	THE EXTENDED COMPLEX PLANE	515
2	LINEAR FRACTIONAL TRANSFORMATIONS	519
	Basic LFTs, 519. General LFTs, 521	
3	CONFORMAL MAPPINGS	524
	Motivation and Definition, 524. More Examples of Conformal Mappings, 527. The Schwarz Lemma and the Riemann Mapping Theorem, 530	
4	SUPPLEMENTARY EXERCISES	534
	Bibliography.	537
	Index.	539